Percussion Synthesis using Loopback Frequency Modulation Oscillators

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ABSTRACT

In this work, we apply recent research results in loopback frequency modulation (FM) to real-time parametric synthesis of percussion sounds. Loopback FM is a variant of FM synthesis whereby the carrier oscillator "loops back" to serve as a modulator of its own frequency. Like FM, more spectral components emerge, but further, when the loopback coefficient is made time varying, frequency trajectories that resemble the nonlinearities heard in acoustic percussion instruments appear. Here, loopback FM is used to parametrically synthesize this effect in struck percussion instruments, known to exhibit frequency sweeps (among other nonlinear characteristics) due to modal coupling. While many percussion synthesis models incorporate such nonlinear effects while aiming for acoustic accuracy, computational efficiency is often sacrificed, prohibiting real-time use. This work seeks to develop a real-time percussion synthesis model that creates a variety of novel sounds and captures the sonic qualities of nonlinear percussion instruments. A linear, modal synthesis percussion model is modified to use loopback FM oscillators, which allows the model to create rich and abstract percussive hits in real-time. Musically intuitive parameters for the percussion model are emphasized resulting in a usable percussion sound synthesizer.

1. INTRODUCTION

Synthesis of plates, membranes, and other percussion instruments have been realized using several different modeling techniques including modal synthesis (MS) [1], the Functional Transformation Method (FTM) [2], finite difference schemes (FDS) [3], and the digital waveguide mesh (DWM) [4]. When real percussion instruments are struck with a large velocity excitation, nonlinear effects often result. Examples of these nonlinearities include the cascade of energy from low to high frequency components that give cymbals their characteristic sound and pitch glides heard with gongs [5].

Many nonlinear percussion synthesis models are computationally expensive and may exhibit stability issues that render them unsuitable for real-time synthesis on standard computers [1–3]. An exception to this is described in [6], where computationally heavy calculations are approximated so that a nonlinear membrane model is able to simulate up to 1000 modes in real-time at a sampling rate of 44.1kHz. For the percussion models in [7] and [2], the most computationally expensive component is the nonlinear calculation. Though the linear version of each model can be efficiently computed, they produce sounds that are less interesting than when nonlinearities are added. With this understanding, one may like to use a linear system to synthesize percussive sounds and approximate the nonlinearities in an efficient and perceptually similar way.

For example, in [2], Avanzini and Marogna present a sound synthesis simulation of a nonlinear, tension-modulated percussion membrane. The model consists of a linear portion and a nonlinear feedback section simulating tension modulation. The linear model is computationally efficient, but the nonlinear tension modulation requires a feedback calculation for every sample, a computational complexity that makes it unable to run in real time. In a following paper [8], the nonlinear feedback calculation is replaced with an efficient approximation that can be calculated with the computational expense similar to that of the linear model.

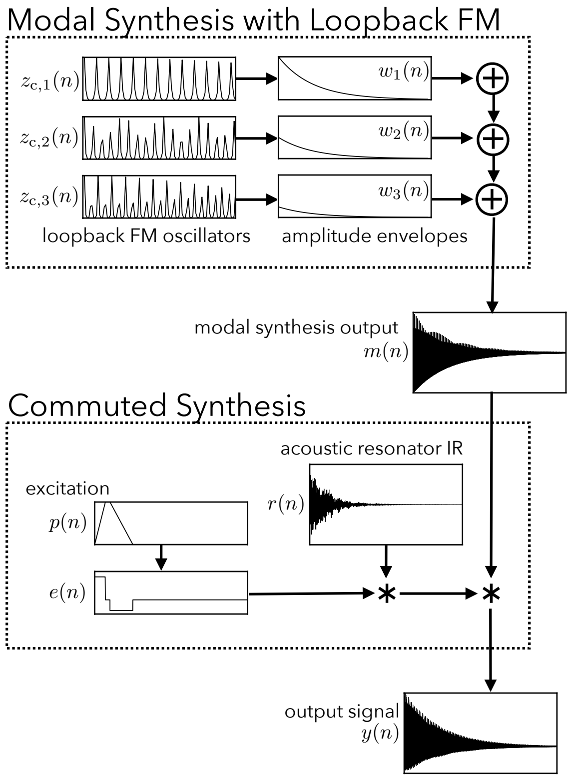

Figure 1. The loopback FM percussion synthesis method.

Instead of striving for an acoustically accurate simulation as some previous research has done, the aim here is to create a percussion synthesizer that creates a variety of novel sounds inspired by the dynamic and nonlinear phenomena heard in percussion instruments. Similar to the strategy used in [9], in which the pitch glide capabilities of a Duffing oscillator are explored in the sound synthesis of a gong, the work presented herein employs the nonlinear effects of loopback FM, a technique initially presented in [10] and further developed in [11]. Here, loopback FM oscillators are used to enhance a modal synthesis (MS) of percussion sound (see Figure 1). Loopback FM is a variant of FM synthesis where the carrier signal is looped back to modulate its own frequency, resulting in complex spectra (much like traditional FM), and interesting frequency trajectories that resemble the nonlinearities observed in real percussion instruments when the loopback coefficient is made time varying.

In Section 2, we explore traditional MS and how it can be used to synthesize percussive sounds. Section 3 reviews loopback FM equations relevant to the current context. Section 4 explains how traditional MS can be modified with loopback FM oscillators to create a wide variety of percussion sounds. Synthesis parameters are discussed in Section 5. Section 6 presents synthesis examples of a marimba, tom tom, and circular plate. Concluding thoughts and future research directions are considered in Section 7.

2. PERCUSSION SYNTHESIS USING TRADITIONAL MODAL SYNTHESIS

MS is a technique that resynthesizes the sound of an acoustic object according to its acoustic modes or vibrational patterns. The resonant frequencies of an acoustic object arise through the sinusoidal motion of the object's modes. With traditional MS, each mode is synthesized using a second-order resonating filter with a corresponding frequency, initial amplitude, and decay [12].

To synthesize a percussive sound with MS, we begin with a list of N_f modal frequencies f_i, the values of which can be obtained from acoustic experiments, spectral analysis of recorded or physically modeled sounds (e.g. DWMs, FDSs, etc), or calculated using theoretical equations.

Though MS traditionally models each frequency mode with a second-order bandpass resonant filter with center frequency f_i, in this work the filters are replaced by sinusoidal oscillators of frequency (or center frequency if frequency is time varying) f_i. This allows for a straightforward comparison with the loopback FM version (also implemented here with oscillators) than if traditional MS bandpass filters had been used. A sinusoidal component with carrier frequency $\omega_{\mathrm{c},i} = 2\pi f_i$ is expressed as $s_i(n) = \sin(\omega_{\mathrm{c},i} nT)$ for time sample n and period $T = 1/f_\mathrm{s}$ for sampling rate f_s. Each sinusoidal component is multiplied with an amplitude envelope $w_i(n)$. For percussive sounds, $w_i(n)$ are typically exponentially decreasing envelopes with possibly different initial amplitudes and decay rates for different modes. For example, for natural sounding results, higher-frequency modes should be made to decay more rapidly. Enveloped sinusoidal components

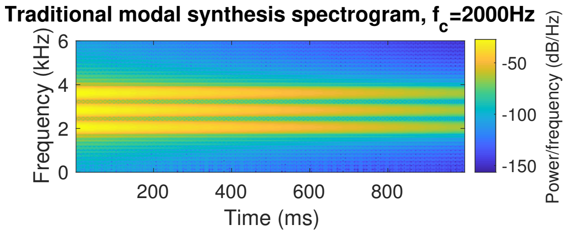

Figure 2. Traditional MS for three modal frequencies.

are added together to form the MS output given by

$$m_s(n) = \sum_{i=0}^{N_\mathrm{f}-1} w_i(n) s_i(n), \tag{1}$$

the spectrum of which is shown in Figure 2 for $N_\mathrm{f} = 3$.

MS is efficient and useful for recreating the sound of objects that consist of a small number of resonant frequency modes. However, MS is a linear method and (1) is incapable of capturing nonlinear effects. A simple modification to the sinusoidal components of the MS framework allows the system to create complex and dynamic sounds reminiscent of nonlinear vibrations in percussion instruments. In this modification, each sinusoidal component is looped back to modulate its own carrier frequency, a synthesis technique coined by the authors as "Loopback FM."

3. LOOPBACK FM

Loopback FM is a self-modulated form of FM where the oscillator loops back and modulates its own carrier frequency according to a feedback coefficient. This differs from Feedback FM [13], in which the output is used to modulate its own initial phase. Loopback FM with a static feedback coefficient, B, and feedback FM both create peaks in the spectrum at integer multiples of a sounding frequency. As described in [10], the difference between the two synthesis methods is that with loopback FM, the feedback coefficient B can be varied over time to create both predictable pitch and spectral changes. Conversely, feedback FM preserves pitch (in some contexts a desirable feature) and only introduces spectral changes. As shown in [11], loopback FM and its closed-form IIR approximation, an expression that resembles the transfer function of a "stretched" allpass filter [14] but for which only the real part is used as a time-domain signal, can be used to create complex frequency spectra and pitch contours. Here, we present the equations for loopback FM and its closed form representation with static pitch and timbre followed by their time-varying formulations, which can be used to modulate timbre and sounding frequency.

3.1 Loopback FM Formulation

The loopback FM equation involves a carrier frequency $\omega_\mathrm{c} = 2\pi f_\mathrm{c}$ where ω_c is the angular frequency and f_c is the frequency in Hz, and a feedback parameter B, which controls the output's timbre and fundamental frequency.

The loopback FM equation for static B and time sample n is

$$z_\mathrm{c}(n) = e^{j\omega_\mathrm{c} T(1 + B\Re\{z_\mathrm{c}(n-1)\})} z_\mathrm{c}(n-1), \tag{2}$$

with the initial condition $z_c(0) = 1$ causing oscillation. The output that we listen to is the real part of $z_c(n)$. The fundamental frequency of this oscillator is not ω_c but rather $\omega_0 = 2\pi f_0$, where f_0 is the sounding frequency in Hz. The relationship between ω_0 and ω_c is described by

$$\omega_0 = \omega_c \sqrt{1 - B^2}, \quad -1 \leq B \leq 1, \tag{3}$$

which shows that for ω_0 to remain real, the value of B must be within the interval $(-1, 1)$.

3.2 An Alternate Representation of the Loopback FM Oscillator

The loopback FM oscillator $z_c(n)$ given in (2) with static pitch and timbre may also be represented by the closed-form representation

$$z_0(n) = \frac{b_0 + e^{j\omega_0 nT}}{1 + b_0 e^{j\omega_0 nT}}, \tag{4}$$

which is similar to the transfer function of a "stretched" allpass filter used in [14]. In this synthesis context, (4) is used as a time-domain signal that is a function of time sample n, where b_0 influences spectrum, ω_0 specifies the sounding frequency, and the sound is the real signal given by $\Re\{z_0(n)\}$. Parameters b_0 and ω_0 are related to the loopback FM feedback coefficient B. With (4), timbre and pitch can be independently controlled, but this is not possible with loopback FM parameters ω_c and B given in (2). Coefficient b_0 in $z_0(n)$ is related to loopback FM parameter B through

$$b_0 = \frac{\sqrt{1 - B^2} - 1}{B}. \tag{5}$$

Note the singularity in (5) for $B = 0$. The relationship between ω_0 and ω_c is shown in (3).

3.3 Time-varying B: Pitch and Timbre Modulation with $z_c(n)$

In (2), the feedback coefficient B can be varied over time between $(-1, 1)$ to create pitch glides and timbre variations over the length of the output signal. From (2), B is replaced by $B(n)$ to form

$$z_c(n) = e^{j\omega_c T(1 + B(n)\Re\{z_c(n-1)\})} z_c(n-1) \tag{6}$$

(3) reveals that when B is made to vary over time, ω_0 also becomes time-varying. This creates a pitch trajectory where the sounding frequency follows

$$\omega_0(n) = \omega_c \sqrt{1 - B^2(n)} \tag{7}$$

3.4 Time-varying b_0 and ω_0: Pitch and Timbre Modulation with $z_0(n)$

Like (6), the parameters of H in (4) can be made to vary over time to create pitch glides and spectral changes. Parameter b_0 can be mapped to $B(n)$ by

$$b_0(n) = \frac{\sqrt{1 - B^2(n)} - 1}{B(n)} \tag{8}$$

and used in (9) to create spectral variations.

A desired pitch contour can be created by setting $\omega_0(n)$ to a pitch trajectory in the form of (7). Directly using $\omega_0(n)$ in place of ω_0 in (4) will not result in the desired pitch glide, and it is necessary to use a generalization of (4):

$$z_0(n) = \frac{b_0(n) + e^{j\Theta_0(n)}}{1 + b_0(n) e^{j\Theta_0(n)}}. \tag{9}$$

To understand $\Theta_0(n)$, the instantaneous phase of the complex exponential terms in (9), let $\omega_0(t)$ be the continuous counterpart of $\omega_0(n)$ serving as the instantaneous frequency, and $\Theta_0(t)$ its integral with respect to time:

$$\Theta_0(t) = \int_0^t \omega_0(t) \mathrm{d}t. \tag{10}$$

Examples of the discrete-time form of (10) given by $\Theta_0(n)$ as used in (9), are shown in Section 5.5.

4. PERCUSSION SYNTHESIS WITH LOOPBACK FM OSCILLATORS

The main steps involved in the loopback FM percussion synthesis method are shown in Figure 1. The "Modal Synthesis with Loopback FM" block consists of synthesizing the vibrations of an abstract, nonlinear surface using MS and (6) or (9) to produce output $m(n)$. The "Commuted Synthesis" block completes the percussion model by convolving a parametric excitation function and acoustic resonator impulse response with $m(n)$.

4.1 Modal Synthesis with Loopback FM

Like the percussion MS technique described in Section 2, the "Modal Synthesis with Loopback FM" block begins with a list of modal frequencies f_i of length N_f. Instead of sinusoidal oscillators, N_f loopback FM oscillators are generated using the frequencies in f_i. As described in [11], the loopback FM oscillators can be expressed as resonating filters, though here, they are implemented as oscillators. This is similar to implementing MS with sinusoidal oscillators as opposed to resonating filters as described in Section 2. The loopback FM oscillator $z_{c,i}(n)$ has been synthesized with carrier frequency $\omega_{c,i} = 2\pi f_i$, where subscript i means that $\omega_{c,i}$ is set using the i^{th} frequency in f_i.

The real part of each loopback FM oscillator is multiplied with an amplitude envelope $w_i(n)$ and the enveloped loopback FM oscillators are summed to create the MS output

$$m(n) = \sum_{i=0}^{N_f - 1} w_i(n) \Re\{z_{c,i}(n)\}. \tag{11}$$

4.2 Commuted Synthesis

In [15], Smith efficiently models stringed musical instruments using commuted synthesis. This technique is adapted here for percussion synthesis.

To complete the percussion instrument model, $m(n)$ must be excited by an excitation function, $e(n)$, and coupled to an acoustic resonator with impulse response $r(n)$. The equation to synthesize this relationship is

$$y(n) = e(n) * m(n) * r(n) \tag{12}$$

where $*$ indicates convolution. Because there is no dependence between $m(n)$, $e(n)$, and $r(n)$, $m(n)$ can be commuted with $r(n)$. The excitation and resonator impulse response can be convolved to form an aggregate excitation $a(n) = e(n) * r(n)$.

Aggregate excitations can be stored for several excitation and resonator combinations. During run-time, a low-latency convolution method, such as the one described in [16], can be used to convolve $a(n)$ with $m(n)$ to form the final percussion model output

$$y(n) = a(n) * m(n). \quad (13)$$

In our syntheses, we use a variety of resonator impulse responses as presented in Section 6 along with two different types of parametric excitations.

4.3 Excitations

The "Excitation" block in Figure 1 involves $p(n)$, a function that describes the vertical position of a drumstick/mallet hitting a surface at time n. The excitation signal $e(n) = p(n) - p(n-1)$, relates to the velocity of the drumstick/mallet and is convolved with the acoustic resonator impulse response to form $a(n)$. Here, we use raised cosine envelopes and filtered noise bursts for $p(n)$. These signals are parametric and affect the resulting output timbre.

4.3.1 Raised Cosine Envelopes

The raised cosine envelope has a single parameter: the window length L. The equation for the excitation is

$$p(n) = \begin{cases} 0.5\left(1 - \cos\left(\dfrac{2\pi n}{L-1}\right)\right), & \text{for } 0 \le n < L \\ 0, & \text{for } n \ge L \end{cases} \quad (14)$$

4.3.2 Filtered Noise Bursts

The parameters for a filtered noise burst are noise burst duration t_d and low and high frequency cutoffs for a bandpass filter f_low and f_high. Examples in this paper use white noise filtered by a second-order Butterworth bandpass filter.

5. MUSICAL PARAMETERS FOR LOOPBACK FM PERCUSSION SYNTHESIS

Musical parameters for the loopback FM percussion synthesis method are presented here along with their corresponding variables and equations.

5.1 Timbre: Oscillators created with $z_{\mathrm{c},i}(n)$ or $z_{0,i}(n)$

The MS oscillators can be synthesized using $z_{\mathrm{c},i}(n)$ or $z_{0,i}(n)$. To use $z_{0,i}(n)$ oscillators, replace $z_{\mathrm{c},i}(n)$ in (11) with $z_{0,i}(n)$. While both forms produce almost identical results from $f_\mathrm{c} = 0$ Hz to around $f_\mathrm{c} = 2500$ Hz at a sampling rate of 44.1 kHz, when $f_\mathrm{c} > 2500$ Hz, the version that uses $z_{\mathrm{c},i}(n)$ becomes much noisier, due to aliasing. If the sampling rate is increased, the output is the same whether the oscillators are created using $z_{\mathrm{c},i}(n)$ or $z_{0,i}(n)$. Like FM, both $z_{\mathrm{c},i}(n)$ and $z_{0,i}(n)$ produce signals

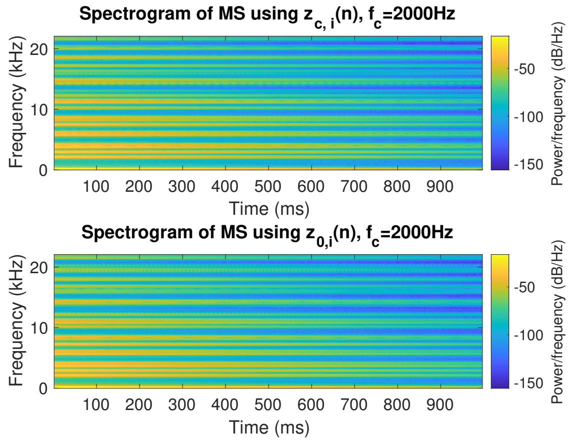

Figure 3. MS using $z_{\mathrm{c},i}(n)$ and $z_{0,i}(n)$ with low carrier frequencies create almost identical results.

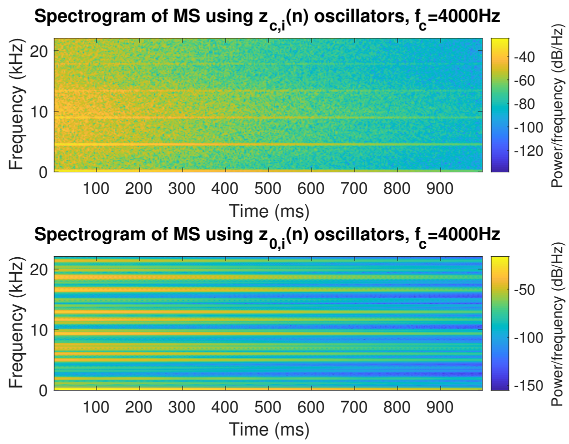

Figure 4. MS with $z_{\mathrm{c},i}(n)$ creates noisier output than MS with $z_{0,i}(n)$ for high carrier frequencies.

that are not bandlimited. With loopback FM, a large f_c means a large feedback amount, which can mean increased bandwidth and aliasing, similar to how a large index of modulation corresponds to a wider bandwidth in traditional FM. Figure 3 shows that the $z_{\mathrm{c},i}(n)$ and $z_{0,i}(n)$ MS oscillators produce similar spectrograms when the lowest of 3 modal frequencies is set to a low carrier frequency of $f_\mathrm{c} = 2000$Hz. Vastly different spectrograms are produced when the lowest of the 3 modal frequencies is set to a higher carrier frequency of $f_\mathrm{c} = 4000$ Hz as shown in Figure 4. The MS using $z_{\mathrm{c},i}(n)$ synthesizes a noisier output and can be used to create cymbal- and crash-like sounds as shown in Section 6.3.

5.2 Timbre: B and b_0

Loopback FM parameter B controls timbre in (2) while $z_0(n)$ parameter b_0 affects timbre in (4). For carrier frequencies below 2500Hz, the frequency components created using (2) or (4) are almost identical and are spaced at integer multiples of f_0. When $B = 0$ or $b_0 = 0$, there are no sidebands and the output is a pure tone. As B and b_0 increase towards 1 (or decrease towards -1), more sidebands appear and the timbre brightens. The sidebands logarithmically decrease in amplitude for each multiple of f_0.

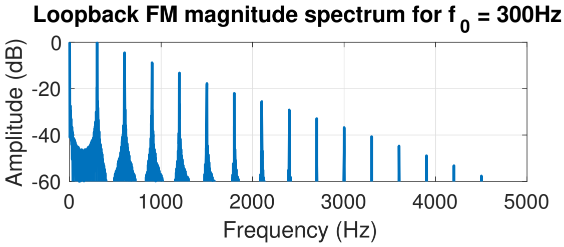

Figure 5. The loopback FM magnitude spectrum. Frequency components occur at integer multiples of the sounding frequency, 300Hz, and the amplitude of the components decreases logarithmically.

Figure 5 is a plot of the magnitude spectrum for a loopback FM oscillator with static $B = 0.9$ and $f_0 = 300$Hz. The sounding frequency can be seen as a peak at 300Hz and the sidebands are spaced at integer multiples of 300Hz with a logarithmic decrease.

(5) explains the relationship between b_0 and B, though as described in Section 5.1, at high carrier frequencies, the output from (2) will differ from that of (4).

5.3 Time-varying Timbre: $B(n)$ and $b_0(n)$

With (6), $B(n)$ affects the time-varying timbre and sounding frequency. When using (9), $b_0(n)$ controls the time-varying timbre, independent of pitch. As in the static case, as $B(n)$ and $b_0(n)$ near 0, the output approaches a pure tone, while as $B(n)$ and $b_0(n)$ approach 1 and -1, the number of sidebands created by the oscillators increases and the timbre becomes brighter.

In Figure 6, $B(n) = g^n$ where $g = 0.9999$, $b_0(n)$ is obtained according to (8), and amplitude envelopes are the same for all modal frequencies. The top and middle plots in Figure 6 compare spectrograms for a static timbre of $b_0 = -0.6312$ with (4) and a time-varying timbre where $b_0(n)$ is used with (9). The sidebands in the top plot are the same over the course of the signal, but the higher frequency sidebands die out over time in the middle plot as $b_0(n)$ increases from -1 to 0. Time-varying timbre between (6) and (9) can be compared using the middle and bottom plots. In the bottom plot, time-varying $B(n)$ creates timbre and pitch variation as n increases. In the middle plot, $b_0(n)$ changes the timbre without affecting the frequency trajectories.

5.4 Sounding Frequency: ω_0

For Eqs. 2 and 4, the sounding frequency can be controlled with $\omega_0 = 2\pi f_0$. For a desired ω_0 with (2), one would use (3) and either 1) set B to a desired value and solve for ω_c or 2) set ω_c and solve for B.

Because the modal frequencies for percussive instruments are often inharmonic, the sounding frequency for percussion synthesis is not clearly defined. With MS using $z_{0,i}(n)$ oscillators, $\omega_{0,i} = 2\pi f_i$ is used to set the sounding frequencies of individual oscillators. For MS using $z_{c,i}(n)$ oscillators, the carrier frequencies can be set to the modal frequencies: $\omega_{c,i} = 2\pi f_i$ or the sounding frequencies can be set to the modal frequencies: $\omega_{0,i} = 2\pi f_i$. According

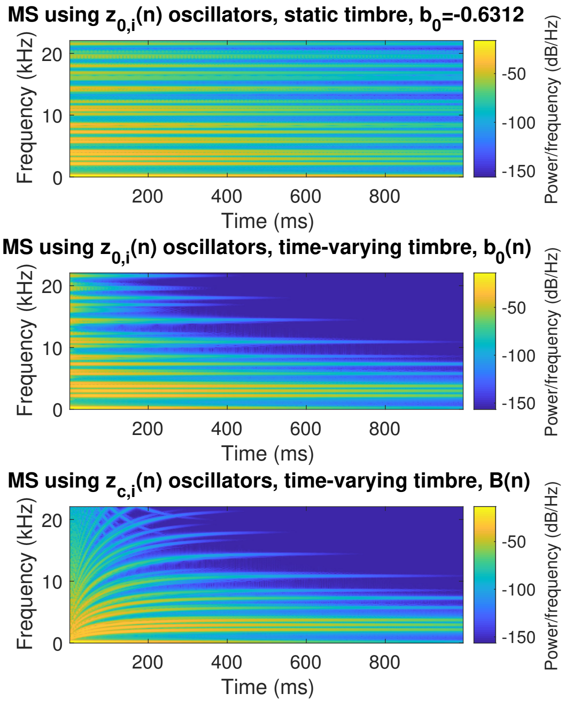

Figure 6. Static and time-varying timbre with various MS oscillators. Middle: $b_0(n)$ with (9) modulates timbre independently of pitch. Bottom: $B(n)$ with (6) affects both timbre and pitch.

to (3), when $B = 0$, $\omega_0 = \omega_c$, and setting either to the modal frequencies would create the same output. When B is large and close to 1 or -1, ω_0 will be a lower frequency than ω_c. This means that using $\omega_{c,i} = 2\pi f_i$ produces lower sounding frequencies while setting $\omega_{0,i} = 2\pi f_i$ produces higher sounding frequencies, which will most likely produce aliasing effects, especially with (2), as described in Section 5.1. Figure 7 demonstrates that when B is close to 1, $\omega_{c,i} = 2\pi f_i$ creates a toned output while $\omega_{0,i} = 2\pi f_i$ creates a noisy output as higher frequencies contribute to extreme aliasing effects.

5.5 Pitch Glides: $B(n)$ and $\omega_0(n)$

With (6), a pitch glide can be added by varying $B(n)$ over time. This also produces timbral changes. A pitch glide can be created with (9) by varying $\omega_0(n)$ over time as described in Section 3.4. To modify the pitch independently of timbre with (9), b_0 should be held constant.

As described in Section 5.1, differences between (6) and (9) can be observed when ω_c is high, and this effect occurs with pitch glides. Figure 8 shows the high carrier frequency difference for a pitch glide over three modal frequencies using MS with (6) and (9). The pitch glide is created with $B(n) = 0.9999^n$, so timbre also changes. At higher carrier frequencies, MS with Eq. 6 creates noise-like output for the first 100ms and more spectral components than MS with Eq. 9 from $100 - 250$ms. From 300ms through the remainder of the signal, the frequency components are more similar.

Pitch glides are constrained to use exponential and linear $B(n)$ functions in the synthesis examples presented in this

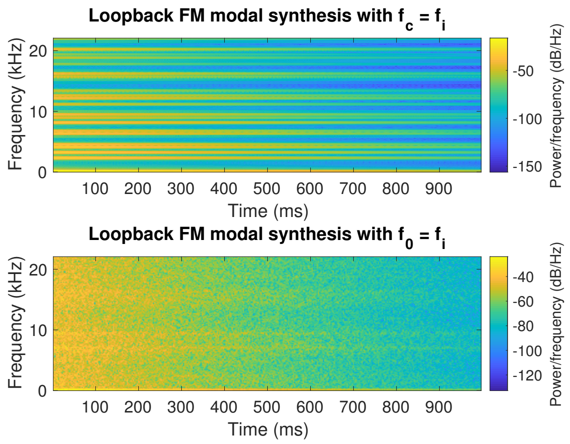

Figure 7. Loopback FM MS spectrograms for $\omega_{c,i} = 2\pi f_i$ (top) and $\omega_{0,i} = 2\pi f_i$ (bottom). The top and bottom signals are generated using the same 3 modal frequencies with $B = 0.9$.

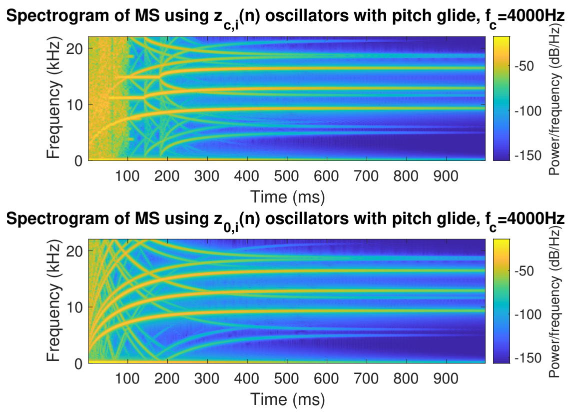

Figure 8. MS using (6) vs. (9) at high carrier frequencies with a pitch glide produce different spectrograms for the first 250ms. The lowest oscillator frequency uses $f_c = 4000$Hz.

research. This restriction on $B(n)$ creates natural sounding pitch glides and allows us to draw parallels between the $z_c(n)$ and $z_0(n)$ forms.

5.5.1 Pitch Glides with Exponential $B(n)$

For the exponential case, $B(n) = g^n$ can be used directly in (6) to produce a pitch glide. When using (9) for a pitch glide, $\Theta_0(n)$ can be found using $B(n) = g^n$ along with Equations 7 and 10 which, as shown in [11], is given by

$$\Theta_0(n) = \frac{\omega_c}{\log(g)}(\sqrt{1-g^{2n}} - \tanh^{-1}(\sqrt{1-g^{2n}})) + C \tag{15}$$

where C is the constant of integration.

5.5.2 Pitch Glides with Linear $B(n)$

For the linear case, $B(n) = kn + l$ produces a pitch glide when used with (6). (9) uses the instantaneous phase given by

$$\Theta_0(n) = \frac{\omega_c}{2k}((kn+l)\sqrt{1-(kn+l)^2} + \sin^{-1}(kn+l)) + C \tag{16}$$

5.6 Decay Time: $w_i(n)$

The decay time for the percussion signal can be controlled through the amplitude envelopes $w_i(n)$. A natural sounding way to set these envelopes is to model them as exponentially decreasing envelopes over time: $w_i(n) = A_0 e^{-n/\tau}$, with different initial amplitude values A_0, as shown in Figure 10, and/or different decay rates τ.

5.7 Commuted Synthesis Parameters

5.7.1 Attack Sharpness: Raised Cosine Envelopes

With raised cosines envelopes, small values of L create sharper sounding attacks, while longer values of L increase the presence of low frequencies in the output and result in bass-heavy sounds. Intuitively, L is proportional to the mass of a hammer or mallet used to excite a drum head: a longer L means a hammer/mallet with greater mass.

5.7.2 Attack Noisiness: Filtered Noise Bursts

For filtered noise burst excitations, a longer noise burst t_d and higher bandpass frequency cutoff f_{high} will create a noisier attack. f_{low} and f_{high} should be tuned to filter out undesired frequencies. For example, for a high pitched percussion sound, the lower frequencies could be filtered out from the noise burst by setting f_{low} to a higher frequency.

5.7.3 Timbre: Acoustic Resonator Impulse Response $r(n)$

The acoustic resonator filters the synthesis output, so the timbre can be further shaped by the frequencies present in $r(n)$. For an expansive and large sound, a room impulse response with a long T60 may work well while for a shorter, tuned sound, the impulse response of a small, acoustic tube model could be used.

6. SYNTHESIS EXAMPLES

While the loopback FM percussion synthesis method is capable of creating a variety of percussive sounds, this section covers three sound synthesis examples that use modal frequencies from [17]: the marimba, tom tom, and circular plate. Although these modal frequencies are associated with real, physical instruments, the aim of this synthesis is not to recreate the naturally occurring sounds. Rather, we seek to synthesize many different types of sounds with nonlinearities similar to those that occur in percussion instruments. For these examples, differences between percussion synthesis using traditional and loopback FM MS are compared for the same modal frequencies, decaying amplitude envelopes, and commuted synthesis parameters. Sound examples can be found at
`http://musicweb.ucsd.edu/~trsmyth/other/percussionSynthesisLoopbackFM.html`.

6.1 Marimba

Figure 9 compares the spectrograms of a marimba modeled as a bar with two free ends using traditional and loopback FM MS. This example sets $\omega_{0,i}$ to seven modal frequencies

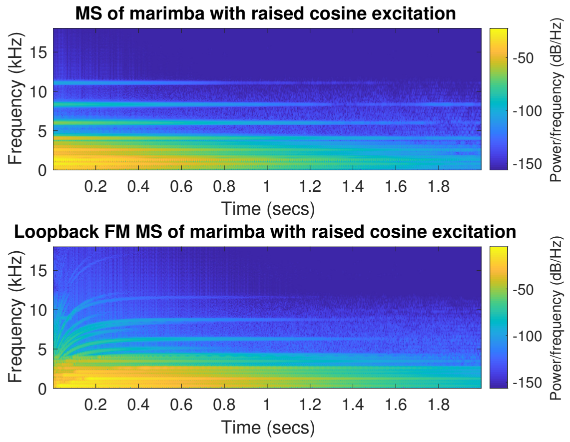

Figure 9. Traditional (top) vs. Loopback FM MS (bottom) using the modal frequencies of an ideal bar with two open ends. The excitation is a raised cosine and the acoustic resonator is an ideal tube synthesized using traditional MS.

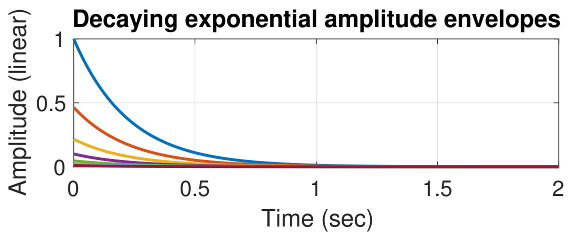

Figure 10. The marimba synthesis amplitude envelopes are decaying exponentials. The initial amplitude of the envelopes is inversely proportional to the modal frequency of the oscillator that is paired with the envelope.

calculated as

$$f_i = \begin{cases} 440, & \text{for } i = 0 \\ 440\frac{(2i+3)^2}{3.011^2}, & \text{otherwise} \end{cases} \quad (17)$$

The amplitude envelopes are decaying exponentials where initial amplitudes decrease exponentially from 1 for the first (lowest) modal frequency to 0.01 for the seventh (highest) modal frequency. Figure 10 is a plot of the amplitude envelopes used for this marimba example. This example is created using $z_{c,i}(n)$ oscillators with an 8-sample length raised cosine excitation and a pitch glide created by setting $B(n) = 0.9999^n$. The acoustic resonator is an ideal, open-closed tube synthesized using traditional MS. Compared to the signal generated using traditional MS, the signal created using loopback FM MS has more frequency components and a clearly increasing pitch glide.

6.2 Tom Tom

The spectrogram of a tom tom synthesized using traditional vs. loopback FM MS is shown in Figure 11. The modal frequencies used to synthesize the tom tom are

$$f_i = 142 \cdot [1, 2.15, 3.17, 3.42, 4.09, 4.80, 4.94] \quad (18)$$

The amplitude envelopes are the same as those used for the marimba as shown in Figure 10. The synthesis uses

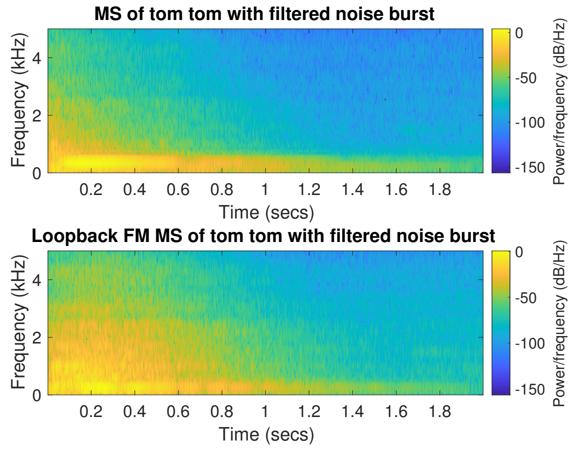

Figure 11. Traditional (top) vs. Loopback FM MS (bottom) using the modal frequencies of a tom tom. The excitation is a filtered noise burst and the acoustic resonator is a taiko drum recording.

$z_{0,i}(n)$ oscillators, $b_0 = -0.9$, and the pitch glide is created linearly increasing B from 0.55 to 0.91. The excitation is a 0.05 second long noise burst filtered with a 2^{nd}-order Butterworth bandpass filter with frequency cutoffs at 120Hz and 4000Hz. The acoustic resonator is a recording of a taiko drum retrieved from `freesound.org`. In Figure 11, there is more high frequency energy for the loopback FM MS than for the traditional MS, especially in the beginning of the signal.

6.3 Circular Plate

In Figure 12, loopback FM MS of a simply-supported circular plate is compared to traditional MS of the same circular plate. The modal frequencies used are

$$f_i = f_0 \cdot [1, 2.80, 5.15, 5.98, 9.75, 14.09, 14.91, 20.66, 26.99] \quad (19)$$

where $f_0 = 0.2287 c_L (h/a^2)$ for plate thickness $h = 0.005$m, plate radius $a = 0.09$m, and longitudinal wave speed $c_L = \sqrt{E/\rho(1-\nu^2)}$ with Young's modulus $E = 2 \cdot 10^{11}\text{N/m}^2$, plate density $\rho = 7860\text{kg/m}^3$, and Poisson ratio $\nu = 0.3$. The amplitude envelopes are decaying exponentials over time. The initial amplitude of these envelopes decreases exponentially as frequency increases from 1 for the lowest modal frequency to 0.5 for the highest modal frequency. Using $z_{c,i}(n)$ oscillators, a slight upwards pitch glide is created by linearly changing $B(n)$ from 0.91 to 0.90 over the course of the signal. The excitation is an 8-sample long raised cosine envelope and the acoustic resonator is a room impulse response retrieved from `echothief.com`. In this example, the drastic aliasing effects in loopback FM MS are used to create an extremely "noisy" signal. Perceptually, the traditional MS output sounds like a clean bell sound, while the loopback FM MS sounds more like a noisy, struck cymbal.

7. CONCLUSIONS

This work has presented a real-time method to synthesize novel, abstract percussion sounds using MS with loopback

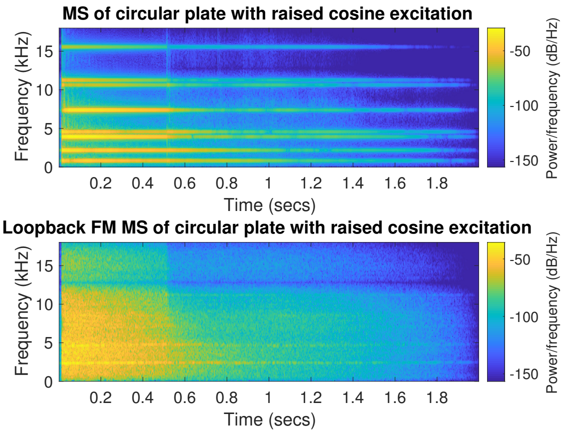

Figure 12. Traditional (top) vs. Loopback FM MS (bottom) using the modal frequencies of a simply-supported circular plate. The excitation is a raised cosine, and the resonator is a room impulse response.

FM oscillators. Loopback FM creates complex spectra and pitch glides similar to the nonlinear effects observed in existing percussion instruments. The synthesis technique allows for parametric control of musical dimensions including sounding frequency, decay time, timbre, and pitch glide. Synthesis examples using the modal frequencies of a marimba, tom tom, and circular plate are examined.

A future research direction involves investigating the aliasing that occurs with large carrier frequencies for both loopback FM and its closed form expression. Another research interest is to explore other methods of creating nonlinearities in oscillators and using these methods with loopback FM to create an even wider range of percussion sounds.

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